ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ then:

$$\frac{ab}{a^2+b^2}+\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{2(a^2+b^2)}\geq\frac{9}{2}$$

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Solution by Tapas Das-India

$$Let\ \frac{a^2+b^2}{ab}=t\overset{AM-GM}{\geq}\frac{2ab}{ab}=2\ (1)$$

$$We\ need\ to\ show:$$

$$\frac{ab}{a^2+b^2}+\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{2(a^2+b^2)}\geq\frac{9}{2}$$

$$or\ \frac{ab}{a^2+b^2}+\frac{a+b}{ab}\sqrt{2.\frac{(a+b)^2}{2}}\overset{CBS}{\geq}\frac{9}{2}$$

$$or\ \frac{ab}{a^2+b^2}+\frac{(a+b)^2}{ab}\geq\frac{9}{2}\ or\ \frac{ab}{a^2+b^2}+\frac{a^2+b^2}{ab}+2\geq\frac{9}{2}$$

$$or\frac{1}{t}+t\overset{\frac{a^2+b^2}{ab}=t}{\geq}\frac{5}{2}\ or\ 2t^2-5t+2\geq0\ or\ (t-2)(2t-1)\geq0\ \ true\ \ by\ \ (1)$$

Equality holds for a=b=1