ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ ab+bc+ca=abc$ then:

$$\frac{a^2}{a+bc}+\frac{b^2}{b+ac}+\frac{c^2}{c+ab}\geq\frac{a+b+c}{4}$$

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Solution by Tapas Das-India

$$a^2+abc\overset{abc=ab+bc+ca}{=}a^2+ab+bc+ca=$$

$$=(a+c)(a+b)\overset{AM-GM}{\leq}\frac{\big((a+b)+(a+c)\big)^2}{4}=\frac{(2a+b+c)^2}{4}\ (1)$$

$$\frac{a^2}{a+bc}+\frac{b^2}{b+ac}+\frac{c^2}{c+ab}=\sum\frac{a^2}{a+bc}=$$

$$=\sum\frac{a^3}{a^2+abc}\overset{(1)}{\geq}\sum\frac{4a^3}{(2a+b+c)^2}\overset{Radon}{\geq}\frac{4(a+b+c)^3}{\big(4(a+b+c)\big)^2}=\frac{a+b+c}{4}$$

Equality holds for a=b=c=3.