ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a}{3a^2+2b^2+c^2}+\frac{a}{3a^2+2b^2+c^2}+\frac{a}{3a^2+2b^2+c^2}\le\frac{1}{6}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

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Solution by Tapas Das-India

$$3a^2+2b^2+c^2=2(a^2+b^2)+(a^2+c^2)\overset{AM-GM}{\ge}4ab+2ac=2a(2b+c)\ (1)$$

$$\frac{a}{3a^2+2b^2+c^2}+\frac{a}{3a^2+2b^2+c^2}+\frac{a}{3a^2+2b^2+c^2}=$$

$$=\sum\frac{a}{3a^2+2b^2+c^2}\overset{(1)}{\le}\frac{1}{2}\sum\frac{1}{2b+c}=$$

$$=\frac{1}{2}\sum\frac{1}{b+b+c}\overset{AM-HM}{\le}\frac{1}{2}\cdot\frac{1}{9}\sum\left(\frac{1}{b}+\frac{1}{b}+\frac{1}{c}\right)=\frac{1}{18}\cdot3\cdot\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{1}{6}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Equality holds for a =b=c.