ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$, $a+b+c\leq 3$ then:

$$\frac{a}{b^{2025}}+\frac{b}{c^{2025}}+\frac{c}{a^{2025}}\geq a+b+c$$

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Solution by Tapas Das-India

$$\frac{a}{b^{2025}}+\frac{b}{c^{2025}}+\frac{c}{a^{2025}}=\sum\frac{a}{b^{2025}}=\sum\frac{a^{2026}}{(ab)^{2025}}\overset{Radon}{\geq}$$

$$\geq\frac{(a+b+c)^{2026}}{(ab+bc+ca)^{2025}}\geq\frac{(a+b+c)^{2026}}{\left(\frac{(a+b+c)^2}{3}\right)^{2025}}=\frac{(a+b+c).(a+b+c)^{2025}.3^{2025}}{(a+b+c)^{2025}.(a+b+c)^{2025}}=$$

$$=(a+b+c).\frac{3^{2025}}{(a+b+c)^{2025}}\overset{a+b+c\leq 3}{\geq}(a+b+c).\frac{3^{2025}}{3^{2025}}=(a+b+c)$$

Equality holds for a=b=c=1.