ROMANIAN MATHEMATICAL MAGAZINE

If $If\ a,b,c>0\ ,a+b+c\geq\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ then:

$$\frac{a^2}{\sqrt{b+8}}+\frac{b^2}{\sqrt{c+8}}+\frac{c^2}{\sqrt{a+8}}\geq 1$$

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Solution by Tapas Das-India

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\overset{C-S}{\geq}9\ or\ (a+b+c)^2\overset{a+b+c\geq\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{\geq}9\ or$$
$$a+b+c\geq 3\ (1)$$

$$24=8\cdot 3\overset{(1)}{\leq}8(a+b+c)\ (2)$$

$$\frac{a^2}{\sqrt{b+8}}+\frac{b^2}{\sqrt{c+8}}+\frac{c^2}{\sqrt{a+8}}\overset{CBS}{\geq}\frac{(a+b+c)^2}{\sqrt{b+8}+\sqrt{c+8}+\sqrt{a+8}}\overset{CBS}{\geq}$$

$$\geq\frac{(a+b+c)^2}{\sqrt{3(a+b+c+24)}}\overset{(2)}{\geq}\frac{(a+b+c)^2}{\sqrt{3((a+b+c+8(a+b+c))}}=\frac{(a+b+c)^{\frac{3}{2}}}{\sqrt{27}}\overset{(1)}{\geq}\frac{(3)^{\frac{3}{2}}}{(3)^{\frac{3}{2}}}=1$$

Equality holds for a=b=c=1.