ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ abc(a+b+c)\geq 3$ then:

$$a^{2026}+b^{2026}+c^{2026}\geq a^{2025}+b^{2025}+c^{2025}$$

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Solution by Tapas Das-India

$$abc(a+b+c)\geq 3$$

$$(ab).(ac)+(ab)(bc)+(bc)(ac)\geq 3$$

$$\frac{(ab+bc+ac)^2}{3}\geq 3$$

$$(ab+bc+ac)^2\geq 9$$

$$\sum ab\geq 3$$

$$\frac{(\sum a)^2}{3}\geq 3$$

$$\sum a\geq 3\ (1)$$

$$a^{2026}+b^{2026}+c^{2026}=a^{2025+1}+b^{2025+1}+c^{2025+1}\overset{Chebyshev}{\geq}$$

$$\geq\frac{1}{3}(a+b+c)\left(a^{2025}+b^{2025}+c^{2025}\right)\overset{(1)}{\geq}a^{2025}+b^{2025}+c^{2025}$$

Equality holds for a=b=c=1.