ROMANIAN MATHEMATICAL MAGAZINE

If $0 < a, b, c < \frac{3}{2}$ and $abc = (3-2a)(3-2b)(3-2c)$

then prove that : $abc \leq 1$

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Let us assume that : $abc > 1$ and then :

$$\sqrt[2]{(3-2a)(3-2b)(3-2c)} > 1 \overset{\text{via A-G}}{\Rightarrow} \frac{1}{3}\sum_{cyc}(3-2a) > 1$$

$$\Rightarrow 9-3 > 2\sum_{cyc} a \Rightarrow 3 > \sum_{cyc} a \overset{A-G}{\geq} 3\sqrt[3]{abc} \Rightarrow abc < 1,$$

a contradiction $\Rightarrow$ our assumption is incorrect $\therefore abc \leq 1$

$\forall a, b, c \in \left(0, \frac{3}{2}\right)$ and $abc = (3-2a)(3-2b)(3-2c)$ (QED)