# ROMANIAN MATHEMATICAL MAGAZINE

**If $a+b+c=a^3+b^3+c^3-3abc=2$, then prove that:**

$$max\{a,b,c\}-min\{a,b,c\}\le\frac{2\sqrt{3}}{3}$$

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We have: $2=a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$, then

$$a^2+b^2+c^2-ab-bc-ca=1,$$

WLOG, we assume that $a\ge b\ge c$. We will prove that

$$a-c\le 2\sqrt{\frac{a^2+b^2+c^2-ab-bc-ca}{3}}.\quad (1)$$

$$RHS_{(1)}=\sqrt{\frac{2}{3}[(a-b)^2+(b-c)^2+(c-a)^2]}\ge\sqrt{\frac{2}{3}\left[\frac{[(a-b)+(b-c)]^2}{2}+(c-a)^2\right]}=a-c,$$

Equality holds iff $a=\frac{2+\sqrt{3}}{3},b=\frac{2}{3},c=\frac{2-\sqrt{3}}{3}$ *and permutation*.