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If $a,b,c>0$ and $\lambda\geq 0$ then :

$$\sum_{cyc}\frac{a}{b+\lambda c}\geq\frac{4}{\lambda+1}-\frac{\sum_{cyc}ab}{(\lambda+1)\sum_{cyc}a^2}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{a}{b+\lambda c}=\sum_{cyc}\frac{a^2}{ab+\lambda ca}\overset{\text{Bergstrom}}{\geq}\frac{\left(\sum_{cyc}a\right)^2}{(\lambda+1)\sum_{cyc}ab}$$

$$\Rightarrow\sum_{cyc}\frac{a}{b+\lambda c}+\frac{\sum_{cyc}ab}{(\lambda+1)\sum_{cyc}a^2}\geq\frac{\left(\sum_{cyc}a\right)^2}{(\lambda+1)\sum_{cyc}ab}+\frac{\sum_{cyc}ab}{(\lambda+1)\sum_{cyc}a^2}$$

$$=\frac{1}{\lambda+1}\cdot\left(\frac{m+2n}{n}+\frac{n}{m}\right)\left(m=\sum_{cyc}a^2,n=\sum_{cyc}ab\right)=\frac{1}{\lambda+1}\cdot\left(\frac{(m+n)^2}{mn}\right)$$

$$\overset{AM-GM}{\geq}\frac{4}{\lambda+1}\Rightarrow\sum_{cyc}\frac{a}{b+\lambda c}\geq\frac{4}{\lambda+1}-\frac{\sum_{cyc}ab}{(\lambda+1)\sum_{cyc}a^2}\ \forall a,b,c>0,\lambda\geq 0,$$

$''='' \text{ iff } a=b=c\ (\text{QED})$