ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1, \lambda \geq 0$ then:

$$\sum \frac{bc}{a^2(b+\lambda c)} \geq \frac{ab+bc+ca}{\lambda+1}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{bc}{a^2(b+\lambda c)} = \sum \frac{abc}{a^3(b+\lambda c)} \overset{abc=1}{=} \sum \frac{\left(\frac{1}{a}\right)^3}{(b+\lambda c)} \overset{Holder}{\geq}$$

$$\geq \frac{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^3}{3(\lambda+1)(a+b+c)} = \frac{\left(\frac{ab+bc+ca}{abc}\right)^3}{3(\lambda+1)(a+b+c)} \overset{abc=1}{=}$$

$$= \frac{(ab+bc+ca)^3}{3(\lambda+1)(a+b+c)} = \frac{ab+bc+ca}{3(\lambda+1)} \cdot \frac{(ab+bc+ca)^2}{(a+b+c)} \geq$$

$$\geq \frac{ab+bc+ca}{3(\lambda+1)} \cdot \frac{3(ab.bc+bc.ca+ca.ab)}{(a+b+c)} =$$

$$= \frac{ab+bc+ca}{3(\lambda+1)} \cdot \frac{3abc(a+b+c)}{(a+b+c)} \overset{abc=1}{=} \frac{ab+bc+ca}{\lambda+1}$$

Equality holds for $a=b=c=1$.