ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ a^2+b^2+c^2=3\ ,\lambda\geq 0$ then:

$$\sum\frac{a^3}{a+\lambda b}\geq\sum\frac{a^2}{\lambda+a^2}$$

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*Solution by Tapas Das-India*

$$\sum\frac{a^2}{\lambda+a^2}=\sum\left(1-\frac{\lambda}{\lambda+a^2}\right)=3-\lambda\sum\frac{1}{\lambda+a^2}\overset{CBS}{\leq}$$
$$\leq 3-\frac{(1+1+1)^2}{3\lambda+a^2+b^2+c^2}\overset{a^2+b^2+c^2=3}{=}3-\frac{9\lambda}{3\lambda+3}=\frac{3}{\lambda+1}\ (A)$$

$$\sum\frac{a^3}{a+\lambda b}=\sum\frac{a^4}{a^2+\lambda ab}\overset{CBS}{\geq}\frac{(a^2+b^2+c^2)^2}{a^2+b^2+c^2+\lambda(ab+bc+ca)}\geq$$
$$\geq\frac{9}{3+\lambda(a^2+b^2+c^2)}\overset{a^2+b^2+c^2=3}{=}\frac{9}{3+3\lambda}=\frac{3}{\lambda+1}\ (B)$$

$$By\ (A)\ \&\ (B) we\ have:$$
$$\sum\frac{a^3}{a+\lambda b}\geq\sum\frac{a^2}{\lambda+a^2}$$

Equality holds for $a=b=c=1$.