ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$, $xyz=1,\ n\in N$, $n\geq 2$, $\lambda\geq 0$ then:

$$\sum\frac{x^n}{x+y+\lambda}\geq\frac{3}{\lambda+2}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$3\lambda=\lambda\cdot 3\cdot(1)^{\frac{1}{3}}\overset{xyz=1}{=}\lambda\cdot 3(xyz)^{\frac{1}{3}}\overset{AM-GM}{\leq}\lambda(x+y+z)\ (1)$$

$$\left(\sum\frac{x^n}{x+y+\lambda}\right)\left(\sum(x+y+\lambda)\right)(1+1+1)^{n-2}\overset{Holder}{\geq}\frac{(x+y+z)^n}{3^{n-2}(\sum(x+y+\lambda))}=$$

$$=\frac{(x+y+z)^n}{3^{n-2}\left(3\lambda+2(x+y+z)\right)}\overset{(1)}{\geq}\frac{(x+y+z)^n}{3^{n-2}\left(\lambda(x+y+z)+2(x+y+z)\right)}=$$

$$=\frac{(x+y+z)^{n-1}}{3^{n-2}(\lambda+2)}\overset{AM-GM}{\geq}\frac{3^{n-1}(xyz)^{\frac{n-1}{3}}}{3^{n-2}(\lambda+2)}=\frac{3}{\lambda+2}\ (as\ xyz=1)$$

Equality holds for $x=y=z$.