ROMANIAN MATHEMATICAL MAGAZINE

$$\text{If } x,y,z>0, x+y+z=1 \text{ and } \lambda\le\frac{3}{2} \text{ then :}$$

$$\sum_{cyc}(x^3+yz)+\lambda xyz\ge\frac{\lambda+12}{27}$$

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$$\sum_{cyc}(x^3+yz)+\lambda xyz-\frac{\lambda+12}{27}\overset{x+y+z=1}{=}$$

$$\sum_{cyc}x^3+\left(\sum_{cyc}x\right)\left(\sum_{cyc}xy\right)-\frac{4}{9}\left(\sum_{cyc}x\right)^3-\frac{\lambda}{27}\left(\left(\sum_{cyc}x\right)^3-27xyz\right)$$

$$\overset{\lambda\le\frac{3}{2}}{\ge}\sum_{cyc}x^3+\left(\sum_{cyc}x\right)\left(\sum_{cyc}xy\right)-\frac{4}{9}\left(\sum_{cyc}x\right)^3-\frac{\left(\frac{3}{2}\right)}{27}\left(\left(\sum_{cyc}x\right)^3-27xyz\right)$$

$$=\frac{1}{18}\left(\sum_{cyc}x^3+3xyz-\left(\sum_{cyc}x^2y+\sum_{cyc}xy^2\right)\right)\overset{Schur}{\ge}0$$

$$\therefore\sum_{cyc}(x^3+yz)+\lambda xyz\ge\frac{\lambda+12}{27}\ \forall\, x,y,z>0\,\big|\,x+y+z=1 \text{ and } \lambda\le\frac{3}{2},$$

$$''='' \text{ iff } x=y=z=\frac{1}{3}\ (\text{QED})$$