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If $a,b,c>0, abc=1$ and $\lambda\geq 0, n\in\mathbb{N}$, then :

$$\sum_{cyc}\frac{bc}{a^{n+1}(b+\lambda c)}\geq\frac{3}{\lambda+1}\left(\frac{\sum_{cyc}bc}{3}\right)^{n}$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{bc}{a^{n+1}(b+\lambda c)}\overset{abc=1}{=}\sum_{cyc}\frac{(bc)^{n+2}}{3^{n}(b+\lambda c)}\overset{Holder}{\geq}\frac{\left(\sum_{cyc}bc\right)^{n+2}}{3^{n}(\lambda+1)\left(\sum_{cyc}a\right)}=$$

$$\geq\frac{\left(\sum_{cyc}bc\right)^{n}.\left(3abc\sum_{cyc}a\right)}{3^{n}(\lambda+1)\left(\sum_{cyc}a\right)}\overset{abc=1}{=}\frac{3}{\lambda+1}\left(\frac{\sum_{cyc}bc}{3}\right)^{n},''='' \text{ iff } a=b=c=1\ (QED)$$