ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,t>0$ and $\lambda\geq 1$, then :

$$\frac{x^2+ty}{t+\lambda y}+\frac{y^2+tx}{t+\lambda x}\geq\frac{x+y}{\lambda}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{x^2+ty}{t+\lambda y}+\frac{y^2+tx}{t+\lambda x}\geq\frac{x+y}{\lambda}\overset{\text{simplifying}}{\Leftrightarrow}$$

$$\lambda^2(x^3+y^3-x^2y-xy^2+2txy)+\lambda(t^2x+t^2y-2txy)-t^2x-t^2y\geq 0$$

$$\Leftrightarrow \lambda^2(x-y)^2+2\lambda txy(\lambda-1)+t^2x(\lambda-1)+t^2y(\lambda-1)\geq 0\rightarrow \text{true}\because x,y,t>0$$

$$\text{and }\lambda\geq 1\therefore\frac{x^2+ty}{t+\lambda y}+\frac{y^2+tx}{t+\lambda x}\geq\frac{x+y}{\lambda}\ \forall x,y,t>0\text{ and }\lambda\geq 1,$$

$$''='' \text{ iff } x=y \text{ and } \lambda=1\ (\text{QED})$$