ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, a+b+c=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ then :

$$\sum_{cyc}\sqrt{\frac{a^2+2}{3}}\le a+b+c$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\sqrt{\frac{a^2+2}{3}}\overset{CBS}{\le}\sqrt{3}.\sqrt{\sum_{cyc}\frac{a^2+2}{3}}=\sqrt{\sum_{cyc}a^2+6}\overset{?}{\le}\sum_{cyc}a$$

$$\Leftrightarrow\sum_{cyc}a^2+6\overset{?}{\le}\sum_{cyc}a^2+2\sum_{cyc}ab\Leftrightarrow\sum_{cyc}ab\overset{?}{\ge}3\to true\because\sum_{cyc}ab=abc\sum_{cyc}a\le$$

$$\frac{1}{3}\left(\sum_{cyc}ab\right)^2\Rightarrow\sum_{cyc}ab\ge 3\therefore\sum_{cyc}\sqrt{\frac{a^2+2}{3}}\le a+b+c$$

$$\forall a,b,c>0\,|\,a+b+c=\frac{1}{a}+\frac{1}{b}+\frac{1}{c},''='' \text{ iff } a=b=c=1\ (QED)$$