ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $a^3+b^3+c^3+3abc=6$ then:

$$\sum \frac{a^5+2}{a^2}\geq 9$$

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Solution by Tapas Das-India

$$a^3+b^3+c^3+3abc=6 \ or \ 3abc+3abc \overset{AM-GM}{\leq} 6 \ or \ abc\leq 1$$

$$\sum \frac{a^5+2}{a^2}=\sum a^3+2\sum \frac{1}{a^2} \overset{a^3+b^3+c^3+3abc=6 \ \& AM-GM}{\geq}$$

$$\geq 6-3abc+\frac{6}{abc} \overset{abc\leq 1}{\geq} 6-3\cdot 1+\frac{6}{1}=9$$

Equality holds for $a=b=c=1$.