ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\sum a^{3n+1}\frac{b+c}{bc}\geq 6a^nb^nc^n$$

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$$WLOG\ a\geq b\geq c\ then\frac{1}{a}\leq\frac{1}{b}\leq\frac{1}{c}$$

$$\sum a^{3n+1}\frac{b+c}{bc}=\sum a^{3n+1}\left(\frac{1}{b}+\frac{1}{c}\right)\overset{Chebyshev}{\geq}$$

$$\geq\frac{1}{3}\left(\sum a^{3n+1}\right)\sum\left(\frac{1}{b}+\frac{1}{c}\right)=\frac{1}{3}\left(\sum a^{3n+1}\right)\cdot 2\sum\frac{1}{a}\overset{AM-GM}{\geq}$$

$$\geq\frac{2}{3}\cdot 3(abc)^{\frac{3n+1}{3}}\cdot\frac{3}{(abc)^{\frac{1}{3}}}=6a^nb^nc^n$$

Equality case: $a=b=c=1$.