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If $a,b,c>0$ then:

$$\sum a^4\frac{b+c}{bc}\geq 6abc$$

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Solution by Tapas Das-India

$$WLOG\ a\geq b\geq c\ then\frac{1}{c}\geq\frac{1}{b}\geq\frac{1}{a}$$

$$\sum a^4\frac{b+c}{bc}=\sum a^4\left(\frac{1}{b}+\frac{1}{c}\right)\overset{Chebyshev}{\geq}\frac{1}{3}\left(\sum a^4\right)\left(\sum\left(\frac{1}{b}+\frac{1}{c}\right)\right)\geq$$

$$\geq\frac{1}{3}\left(\sum a^4\right)\cdot 2\sum\frac{1}{a}\overset{AM-GM}{\geq}\frac{2}{3}\cdot 3(abc)^{\frac{4}{3}}\cdot 3\left(\frac{1}{abc}\right)^{\frac{1}{3}}=3abc$$

Equality holds for $a=b=c=1$.