ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\sum\frac{(a^2+\lambda bc)(b^2+\lambda ac)}{a^2b^2}\geq 3(1+\lambda)^2$$

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$$\frac{(a^2+\lambda bc)(b^2+\lambda ac)}{a^2b^2}=\frac{c^2}{ab}\left(\frac{a^2+\lambda bc}{bc}\right)\left(\frac{b^2+\lambda ac}{ac}\right)=$$

$$=\frac{c^2}{ab}\left(\frac{a^2}{bc}+\lambda\right)\left(\frac{b^2}{ac}+\lambda\right)\overset{C-S}{\geq}\frac{c^2}{ab}\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2$$

$$\sum\frac{(a^2+\lambda bc)(b^2+\lambda ac)}{a^2b^2}\geq\sum\frac{c^2}{ab}\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2\overset{AM-GM}{\geq}$$

$$\geq 3\sqrt[3]{\prod\frac{c^2}{ab}\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2}=3\left(\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2\left(\frac{\sqrt{ab}}{c}+\lambda\right)^2\right)^{\frac{1}{3}}\overset{Holder}{\geq}$$

$$\geq 3\left(1^{\frac{1}{3}}+(\lambda^3)^{\frac{1}{3}}\right)^{3\times\frac{2}{3}}\geq 3(1+\lambda)^2$$

Equality holds for $a=b=c=1$.