ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0, x+y+z=3$ then:

$$\frac{2x}{x^6+y^4}+\frac{2y}{y^6+z^4}+\frac{2z}{z^6+x^4}\le\frac{1}{x^4}+\frac{1}{y^4}+\frac{1}{z^4}$$

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Solution by Tapas Das-India

$$\frac{2x}{x^6+y^4}+\frac{2y}{y^6+z^4}+\frac{2z}{z^6+x^4}\overset{AM-GM}{\le}\sum\frac{2x}{2x^3y^2}=$$

$$=\sum\frac{1}{x^2y^2}=\frac{x^2+y^2+z^2}{x^2y^2z^2}=\frac{x^2y^2z^2(x^2+y^2+z^2)}{x^4y^4z^4}=$$

$$=\frac{\sum(x^2y^2)(x^2z^2)}{x^4y^4z^4}\overset{(\sum ab)\le(\sum a^2)\forall\, a,b,c>0}{\le}$$

$$\le\frac{x^4y^4+y^4z^4+z^4x^4}{x^4y^4z^4}=\frac{1}{x^4}+\frac{1}{y^4}+\frac{1}{z^4}$$

Equality holds for $x=y=z=1$.