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If $a, b, c > 0$ then:

$$a^{1-\frac{1}{a}} + b^{1-\frac{1}{b}} + c^{1-\frac{1}{c}} \geq 27^{\frac{1}{a+b+c}} \cdot (a+b+c)^{\frac{a+b+c-3}{a+b+c}}$$

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Solution by Ravi Prakash-New Delhi-India

$$\frac{a^{1-\frac{1}{a}} + b^{1-\frac{1}{b}} + c^{1-\frac{1}{c}}}{a+b+c} = \frac{a\left(a^{-\frac{1}{a}}\right) + b\left(b^{-\frac{1}{b}}\right) + c\left(c^{-\frac{1}{c}}\right)}{a+b+c} \geq$$

$$\geq \left[\left(a^{-\frac{1}{a}}\right)^a \left(b^{-\frac{1}{b}}\right)^b \left(c^{-\frac{1}{c}}\right)^c\right]^{\frac{1}{(a+b+c)}} = \frac{1}{(abc)^{\frac{1}{(a+b+c)}}} \quad (1)$$

Also, $(abc)^{\frac{1}{3}} \leq \frac{a+b+c}{3} \Rightarrow \frac{1}{abc} \geq \frac{27}{(a+b+c)^3} \quad (2)$

Thus, from (1) and (2)

$$a^{1-\frac{1}{a}} + b^{1-\frac{1}{b}} + c^{1-\frac{1}{c}} \geq (a+b+c)\left(\frac{27}{(a+b+c)^3}\right)^{\frac{1}{a+b+c}}$$

$$= 27^{\frac{1}{a+b+c}} \cdot (a+b+c)^{1-\frac{3}{a+b+c}} = 27^{\frac{1}{a+b+c}} \cdot (a+b+c)^{\frac{a+b+c-3}{a+b+c}}$$

Equality holds for $a = b = c$.