A NEW GENERALIZATION FOR HADWIGER - FINSLER'S INEQUALITY IN TRIANGLE

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Abstract. In this paper we will give a generalization for Hadwiger - Finsler's inequality.

Main result:
If $m \geq 0$ then in any triangle ABC the following relationship holds:

$$\sum_{cyc} a^{2m+2} \geq 4^{m+1} \cdot (\sqrt{3})^{1-m} \cdot F^{m+1} + \frac{1}{2}(a^{m+1} - b^{m+1}) \tag{1}$$

If $m = a$ then (1) becomes the classical Hadwiger - Finsler's inequality:

$$\sum_{cyc} a^2 \geq 4\sqrt{3}F + \frac{1}{2}\sum_{cyc}(a-b)^2$$

Proof 1.

$$\sum_{cyc}(a^{m+1} - b^{m+1})^2 = 2\sum_{cyc} a^{2m+2} - 2\sum_{cyc}(ab)^{m+1} \Rightarrow$$

$$\sum_{cyc} a^{2m+2} = \sum_{cyc}(ab)^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 \geq$$

$$\overset{\text{RADON}}{\geq} \frac{(ab+bc+ca)^{m+1}}{(1+1+1)^m} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 \geq$$

$$\overset{\text{GORDON}}{\geq} \frac{(4\sqrt{3}F)^{m+1}}{3^m} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 =$$

$$= 4^{m+1} \cdot (\sqrt{3})^{m+1-2m} \cdot F^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 =$$

$$= 4^m \cdot (\sqrt{3})^{1-m} \cdot F^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2$$

□

Proof 2.

$$\sum_{cyc} a^{2m+2} = \sum_{cyc}(ab)^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 \geq$$

$$\overset{\text{AM-GM}}{\geq} 3 \cdot (\sqrt[3]{ab \cdot bc \cdot ca})^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 =$$

$$= \frac{3 \cdot 3^{m+1}}{3^{m+1}} \cdot (\sqrt[3]{(abc)^2})^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 \geq$$

$$\overset{\text{CARLITZ}}{\geq} \frac{3}{3^{m+1}}(4\sqrt{3}F)^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 =$$

$$= \frac{1}{(\sqrt{3})^{2m}} \cdot 4^{m+1} \cdot (\sqrt{3})^{m+1} \cdot F^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2 =$$

$$= 4^{m+1} \cdot (\sqrt{3})^{1-m} \cdot F^{m+1} + \frac{1}{2}\sum_{cyc}(a^{m+1} - b^{m+1})^2$$

Equality holds for $a = b = c$. □

References

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