ROMANIAN MATHEMATICAL MAGAZINE

Prove that:

$$44 < \sqrt{2025 + \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}}} < 47$$

Proposed by Nguyen Van Canh-Vietnam

Solution by Tapas Das-India

$$4^5 = 1024 \,\&\, 5^5 = 3125 \; then \; 1024 < 2028 < 3125 \; or, 4 < \sqrt[5]{2028} < 5$$

$$2027 + 4 < 2027 + \sqrt[5]{2028} < 2027 + 5 \; or, 2031 < 2027 + \sqrt[5]{2028} < 2032$$

$$6^4 = 1296 \,\&\, 7^4 = 2401 \; then \; 1296 < 2031 < 2032 < 2401$$

$$then \; 1296 < 2027 + \sqrt[5]{2028} < 2401 \; or \; 6 < \sqrt[4]{2027 + \sqrt[5]{2028}} < 7$$

$$2026 + 6 < 2026 + \sqrt[4]{2027 + \sqrt[5]{2028}} < 2026 + 7$$

$$2032 < 2026 + \sqrt[4]{2027 + \sqrt[5]{2028}} < 2033$$

$$12^3 = 1728 \,\&\, 13^3 = 2197 \; and \; 1728 < 2032 < 2033 < 2197$$

$$1728 < 2026 + \sqrt[4]{2027 + \sqrt[5]{2028}} < 2197$$

$$12 < \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}} < 13$$

$$2025 + 12 < 2025 + \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}} < 2025 + 13$$

$$2037 < 2025 + \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}} < 2038$$

$$44^2 = 1936 \,\&\, 47^2 = 2209 \; and \; 1936 < 2037 < 2038 < 2209$$

$$then \; 1936 < 2025 + \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}} < 2209$$

$$44 < \sqrt{2025 + \sqrt[3]{2026 + \sqrt[4]{2027 + \sqrt[5]{2028}}}} < 47$$