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If $A, B \in M_n(\mathbb{R}), AB + A + B = O_n$ then:

$$det^2(A + I_n) + det^2(B + I_n) \geq 2$$

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$$det^2(A + I_n) + det^2(B + I_n) = (|det(A + I_n)|)^2 + (|det(B + I_n)|)^2 \geq$$

$$\overset{AM-GM}{\geq} 2\sqrt{|det(A + I_n)| \cdot |det(B + I_n)|} = 2\sqrt{\left|det\left((A + I_n)(B + I_n)\right)\right|} =$$

$$= 2\sqrt{|det(AB + A + B + I_n)|} = 2\sqrt{detI_n} = 2 \cdot \sqrt{1} = 2$$

Equality holds for $|det(A + I_n)| = |det(B + I_n)|$.