

ROMANIAN MATHEMATICAL MAGAZINE

If $0 < x < \frac{\pi}{2}$ then:

$$\sin(\cos x) < \cos(\sin x)$$

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$$\sin x + \cos x = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right) = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) \sqrt{2} \quad (1)$$

$$\left(\text{as } \sin \left(x + \frac{\pi}{4} \right) \leq 1 \right)$$

$$\text{Now: } \sqrt{2} = 1.414(\text{app}) \text{ and } \frac{\pi}{2} = \frac{1}{2} \cdot \frac{22}{7} = \frac{11}{7} = 1.57(\text{app})$$

$$\text{So } \sqrt{2} < \frac{\pi}{2} \text{ or } \sin x + \cos x \stackrel{(1)}{<} \frac{\pi}{2} \text{ or } \cos x < \frac{\pi}{2} - \sin x$$

$$\sin(\cos x) < \sin \left(\frac{\pi}{2} - \sin x \right)$$

$$\sin(\cos x) < \cos(\sin x)$$