ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{2(a^2+b^2)}{b+c}+\frac{c^2}{b}\geq 2a+c$$

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Solution by Tapas Das-India

$$\frac{(a+b)^2}{b+c}+(b+c)\overset{AM-GM}{\geq} 2\sqrt{\frac{(a+b)^2}{b+c}.(b+c)}=2(a+b)$$

$$or, \frac{(a+b)^2}{b+c}\geq 2(a+b)-(b+c)=2a+b-c\ (1)$$

$$\frac{c^2}{b}+b\overset{AM-GM}{\geq} 2c\ or, \frac{c^2}{b}\geq 2c-b\ (2)$$

$$\frac{2(a^2+b^2)}{b+c}+\frac{c^2}{b}\overset{CBS}{\geq}\frac{2\frac{(a+b)^2}{2}}{b+c}+\frac{c^2}{b}=\frac{(a+b)^2}{b+c}+\frac{c^2}{b}\overset{(1)\&(2)}{\geq}$$

$$\geq 2a+b-c+2c-b=2a+c$$

Equality holds for a=b=c.