ROMANIAN MATHEMATICAL MAGAZINE

If $n \in N$ then:

$$\sqrt{4^n \cos^{4n} x + 3} + \sqrt{4^n \sin^{4n} x + 3} \geq 4$$

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Solution by Tapas Das-India

$$\cos^{2n} x + \sin^{2n} x \overset{CBS}{\geq} \frac{(\cos^2 x + \sin^2 x)^n}{2^{n-1}} = \frac{1}{2^{n-1}} \ (1)$$

$$\sqrt{4^n \cos^{4n} x + 3} + \sqrt{4^n \sin^{4n} x + 3} =$$

$$= \sqrt{(2^n \cos^{2n} x)^2 + \left(\sqrt{3}\right)^2} + \sqrt{(2^n \sin^{2n} x)^2 + \left(\sqrt{3}\right)^2} \overset{Minkowski}{\geq}$$

$$\geq \sqrt{(2^n \cos^{2n} x + 2^n \sin^{2n} x)^2 + \left(2\sqrt{3}\right)^2} \overset{(1)}{\geq} \sqrt{2^{2n}.\left(\frac{1}{2^{n-1}}\right)^2 + 12}, = \sqrt{2^2 + 12} = 4$$

$$Equality\ holds\ for\ x = \frac{\pi}{4}$$