ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c \in \mathbb{R}, 2^a + 2^b + 2^c \geq 3^a + 3^b + 3^c$ then:

$$a + b + c \leq 0$$

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By using the Mean Value Theorem for $f(x) = e^{ax}$ on the interval $[ln2, ln3]$, there exists $c \in (ln2, ln3)$ such that

$$3^a - 2^a = f(ln3) - f(ln2) = ln\left(\frac{3}{2}\right) f'(c) = ln\left(\frac{3}{2}\right).ae^{ac} =$$

$$= ln\left(\frac{3}{2}\right).[a + a(e^{ac} - 1)] \geq ln\left(\frac{3}{2}\right).a,$$

because a and $e^{ac} - 1$ have the same sign for all $a \in \mathbb{R}$.

$$\Rightarrow \ 3^a - 2^a \geq ln\left(\frac{3}{2}\right).a \ \text{(and analogs)}$$

$$\Rightarrow 0 \geq (3^a - 2^a) + \left(3^b - 2^b\right) + (3^c - 2^c) \geq ln\left(\frac{3}{2}\right).(a + b + c).$$

Therefore $a + b + c \leq 0$. Equality holds iff $a = b = c = 0$.