ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0, x + y^3 + z^5 \geq x^2 + y^4 + z^6$ then:

$$x + y + z \leq 3$$

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Using the AM − GM inequality, we have

$$3 + x + y^3 + z^5 \geq 3 + x^2 + y^4 + z^6 = (1 + x^2) + \frac{1 + 3y^4}{4} + \frac{3 + y^4}{4} + \frac{1 + 5z^6}{6} + \frac{5 + z^6}{6}$$

$$\geq 2x + \sqrt[4]{1.(y^4)^3} + \sqrt[4]{1^3.y^4} + \sqrt[6]{1.(z^6)^5} + \sqrt[6]{1^5.z^6} = 2x + y^3 + y + z^5 + z.$$

Then $3 \geq x + y + z$. Equality holds iff $x = y = z = 1$.