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If $a, b, c, d, e > 0$, $a + b + c + d + e = 5$ then:

$$4a^3 + 16b^3 + 36c^3 + 324d^3 + 11296e^3 \geq 125$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\begin{aligned} & 4a^3 + 16b^3 + 36c^3 + 324d^3 + 11296e^3 = \\ &= \frac{a^3}{\frac{1}{4}} + \frac{b^3}{\frac{1}{16}} + \frac{c^3}{\frac{1}{36}} + \frac{d^3}{\frac{1}{324}} + \frac{d^3}{\frac{1}{1296}} = \frac{a^3}{\left(\frac{1}{2}\right)^2} + \frac{b^3}{\left(\frac{1}{4}\right)^2} + \frac{c^3}{\left(\frac{1}{6}\right)^2} + \frac{d^3}{\left(\frac{1}{18}\right)^2} + \frac{e^3}{\left(\frac{1}{36}\right)^2} \stackrel{\text{Radon}}{\geq} \\ &\geq \frac{(a+b+c+d+e)^3}{\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{18} + \frac{1}{36}\right)^2} \stackrel{a+b+c+d+e=5}{=} \frac{125}{\left(\frac{18+9+6+2+1}{36}\right)^2} = 125 \end{aligned}$$

Equality holds for $a=b=c=d=e=1$.