ROMANIAN MATHEMATICAL MAGAZINE

If $n \in Z^{+}$ then:

$$0 < \sqrt{4n+2} - \sqrt{n} - \sqrt{n+1} < \frac{1}{16\sqrt[3]{n}}$$

Proposed by Mais Hasanov-Azerbaijan

Solution by Amin Hajiyev-Azerbaijan

$$K = \sqrt{4n+2} - \sqrt{n} - \sqrt{n+1} = \sqrt{4n+2} - \left(\sqrt{n} + \sqrt{n+1}\right)$$

$$K > 0 \rightarrow \sqrt{4n+2} > \sqrt{n} + \sqrt{n+1} \rightarrow 4n+2 > \left(\sqrt{n} + \sqrt{n+1}\right)^2$$

$$4n+2 > 2n+1+2\sqrt{n^2+n} \rightarrow 2n+1 > 2\sqrt{n^2+n} \rightarrow (2n+1)^2 > 4n^2+4n$$

$$4n^2+4n+1 > 4n^2+4n, \quad 1 > 0 \rightarrow K > 0$$

$$K < \frac{1}{16\sqrt[3]{n}} \quad K = \sqrt{4n+2} - \left(\sqrt{n} + \sqrt{n+1}\right)$$

$$K = \frac{\left(\sqrt{4n+2}\right)^2 - \left(\sqrt{n}+\sqrt{n+1}\right)^2}{\sqrt{4n+2}+\sqrt{n}+\sqrt{n+1}} = \frac{2n+1-2\sqrt{n^2+n}}{\sqrt{4n+2}+\sqrt{n}+\sqrt{n+1}} =$$

$$= \frac{(2n+1)^2 - \left(2\sqrt{n^2+n}\right)^2}{\left(\sqrt{4n+2}+\sqrt{n}+\sqrt{n+1}\right)\left(2n+1+2\sqrt{n^2+n}\right)} =$$

$$= \frac{1}{\left(\sqrt{4n+2}+\sqrt{n}+\sqrt{n+1}\right)\left(2n+1+2\sqrt{n^2+n}\right)}$$

$$n \geq 1 \rightarrow \sqrt{4n+2}+\sqrt{n}+\sqrt{n+1} > \sqrt{4n}+\sqrt{n}+\sqrt{n} = 4\sqrt{n}$$

$$2n+1+2\sqrt{n^2+n} > 2n+2\sqrt{n^2} = 4n$$

$$K < \frac{1}{16n\sqrt{n}} \rightarrow n^{\frac{3}{2}} > n^{\frac{1}{3}} \rightarrow \frac{1}{n\sqrt{n}} < \frac{1}{\sqrt[3]{n}}$$

$$0 < K < \frac{1}{16\sqrt[3]{n}}$$