ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$bc+ac+4ab+\frac{48}{3a+4b+5c}>\frac{64}{4a+5b+6c}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$bc+ac+4ab+\frac{48}{3a+4b+5c}=\frac{abc}{a}+\frac{abc}{b}+\frac{4abc}{c}+\frac{16}{\frac{1}{3}(3a+4b+5c)}=$$

$$=\frac{1^2}{a}+\frac{1^2}{b}+\frac{2^2}{c}+\frac{4^2}{\frac{1}{3}(3a+4b+5c)}\overset{Bergstrom}{\geq}\frac{(1+1+2+4)^2}{(a+b+c)+\frac{1}{3}(3a+4b+5c)}=$$

$$=\frac{64}{2a+\frac{7b}{3}+\frac{8c}{3}}=\frac{3.64}{6a+7b+8c}>\frac{3.64}{12a+15b+18c}=\frac{64}{4a+5b+6c}$$