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In ΔABC the following relationship holds:

$$m_a \cot\frac{A}{2} + m_b \cot\frac{B}{2} + m_c \cot\frac{C}{2} \ge \frac{2s^3}{9Rr}$$

Proposed by Nguyen Minh Tho-Vietnam

Solution by Tapas Das-India

$$m_a \cot\frac{A}{2} + m_b \cot\frac{B}{2} + m_c \cot\frac{C}{2} = \sum m_a \cot\frac{A}{2} \overset{m_a \ge \sqrt{s(s-a)}}{\ge}$$

$$\ge \sum \sqrt{s(s-a)}\sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = \sum \frac{s(s-a)}{\sqrt{(s-b)(s-c)}} =$$

$$= s\sum \frac{(s-a)^{\frac{3}{2}}}{\sqrt{(s-b)(s-a)(s-c)}} = \frac{s}{\sqrt{sr^2}}\sum (s-a)^{\frac{3}{2}} =$$

$$= \frac{\sqrt{s}}{r}\sum (s-a)^{\frac{3}{2}} \overset{CBS}{\ge} \frac{\sqrt{s}}{r}\frac{\left(\sum (s-a)\right)^{\frac{3}{2}}}{3^{\frac{1}{2}}} = \frac{\sqrt{s}}{r}\frac{(s)^{\frac{3}{2}}}{3^{\frac{1}{2}}} = \frac{s^2}{r\sqrt{3}}$$

$$= \frac{s^3}{rs\sqrt{3}} \overset{Mitrinovic}{\ge} \frac{s^3}{\frac{r3\sqrt{3}R}{2}\sqrt{3}} = \frac{2s^3}{9Rr}$$

Equality holds for an equilateral triangle.