ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$m_a w_a + m_b w_b + m_c w_c \geq s^2$$

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Solution by Tapas Das-India

Using the known inequalities:

$$m_a \geq \frac{b+c}{2}\cos\frac{A}{2} \text{ and } w_a = \frac{2bc}{b+c}\cos\frac{A}{2}$$

$$m_a w_a \geq \frac{b+c}{2}\cos\frac{A}{2} \cdot \frac{2bc}{b+c}\cos\frac{A}{2} = bc\cos^2\frac{A}{2} = abc\frac{\cos^2\frac{A}{2}}{a} =$$

$$= abc\frac{\cos^2\frac{A}{2}}{4R\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{4Rrs}{4R}\cot\frac{A}{2} = rs\cot\frac{A}{2} \ (1)$$

$$m_a w_a + m_b w_b + m_c w_c = \sum m_a w_a \overset{(1)}{\geq} rs\sum\cot\frac{A}{2} = rs.\frac{s}{r} = s^2$$

Equality holds for an equilateral triangle