ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a}{a^2+bc}+\frac{m_b}{b^2+ca}+\frac{m_c}{c^2+ab}\ge\frac{3}{4R}$$

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Solution by Tapas Das-India

$$\sqrt{(a^2+b^2)(a^2+c^2)}\overset{C-S}{\ge}(a^2+bc)\,,m_a\overset{Tereshin}{\ge}\frac{b^2+c^2}{4R}$$

$$\frac{m_a}{a^2+bc}\ge\frac{\frac{b^2+c^2}{4R}}{\sqrt{(a^2+b^2)(a^2+c^2)}}=\frac{1}{4R}\frac{b^2+c^2}{\sqrt{(a^2+b^2)(a^2+c^2)}}\quad(1)$$

$$\frac{m_a}{a^2+bc}+\frac{m_b}{b^2+ca}+\frac{m_c}{c^2+ab}=\sum\frac{m_a}{a^2+bc}\overset{(1)}{\ge}$$

$$\ge\frac{1}{4R}\sum\frac{b^2+c^2}{\sqrt{(a^2+b^2)(a^2+c^2)}}\overset{AM-GM}{\ge}\frac{3}{4R}\sqrt[3]{\prod\frac{b^2+c^2}{\sqrt{(a^2+b^2)(a^2+c^2)}}}=\frac{3}{4R}$$

Equality holds for an equilateral triangle.