ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \cot^2\frac{A}{2}+3\left(\sum \tan\frac{A}{2}\right)^2 \ge 2\sum \cot\frac{A}{2}\cot\frac{B}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum \cot^2\frac{A}{2}=\left(\sum \cot\frac{A}{2}\right)^2-2\sum \cot\frac{A}{2}\cot\frac{B}{2}=\left(\frac{s}{r}\right)^2-2\sum\frac{s^2}{r_ar_b}=$$

$$=\frac{s^2}{r^2}-2s^2\frac{\left(\sum r_a\right)}{r_ar_br_c}=\frac{s^2}{r^2}-\frac{2s^2(4R+r)}{s^2r}=\frac{s^2}{r^2}-\frac{8R+2r}{r}\ (1)$$

$$2\sum \cot\frac{A}{2}\cot\frac{B}{2}=2\sum\frac{s^2}{r_ar_b}=2s^2\frac{\left(\sum r_a\right)}{r_ar_br_c}=\frac{2s^2(4R+r)}{s^2r}=\frac{8R+2r}{r}\ (2)$$

$$3\left(\sum \tan\frac{A}{2}\right)^2=3\left(\frac{4R+r}{s}\right)^2\overset{Doucet}{\ge}3.3=9\ (3)$$

$$\sum \cot^2\frac{A}{2}+3\left(\sum \tan\frac{A}{2}\right)^2\overset{(1)\&(3)}{\ge}\frac{s^2}{r^2}-\frac{8R+2r}{r}+9\overset{Gerretsen}{\ge}$$

$$=\frac{16Rr-5r^2}{r^2}-\frac{8R+2r}{r}+9=\frac{16R-5r-8R-2r+9r}{r}=$$

$$=\frac{8R+2r}{r}\overset{(2)}{\ge}2\sum \cot\frac{A}{2}\cot\frac{B}{2}$$

$$Equality\ holds\ for\ A=B=C$$