ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum r_a^2 \geq \sum m_a^2$$

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Solution by Tapas Das-India

$$\sum r_a^2 = \left(\sum r_a\right)^2 - 2\sum r_a r_b = (4R+r)^2 - 2s^2 \text{ and}$$

$$\sum m_a^2 = \frac{3}{4}\left(\sum a^2\right) = \frac{3}{4}.2(s^2 - r^2 - 4Rr) = \frac{3s^2 - 3r^2 - 12Rr}{2}$$

$$\text{We need to show}, (4R+r)^2 - 2s^2 \geq \frac{3s^2 - 3r^2 - 12Rr}{2} \text{ or}$$

$$2(4R+r)^2 - 4s^2 \geq 3s^2 - 3r^2 - 12Rr \text{ or}$$

$$2(16R^2 + 8Rr + r^2) + 12Rr + 3r^2 \geq 7s^2 \text{ or}$$

$$32R^2 + 28Rr + 5r^2 \overset{Gerretsen}{\geq} 7(4R^2 + 4Rr + 3r^2) \text{or}$$

$$4R^2 \geq 16r^2 \text{ or } R \geq 2r \text{ true Euler}$$

$$\text{Equality holds for } a = b = c$$