ROMANIAN MATHEMATICAL MAGAZINE

If in $\Delta ABC, c < b, n \in \mathbb{N}$ then:

$$c^n + h_c^n < b^n + h_b^n$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$we\ need\ to\ show, c^n + h_c^n < b^n + h_b^n$$

$$or\ (c^n - b^n) - (h_b^n - h_c^n) < 0$$

$$or\ \ (c^n - b^n) - \frac{1}{2^n R^n}(a^n c^n - a^n b^n) < 0$$

$$or\ \ (c^n - b^n)(2^n R^n - a^n) < 0$$

$$or\ (2^n R^n - a^n) > 0\ (since\ c < b)$$

$$or\ 2^n R^n > a^n = 2^n\ R^n \sin^n A$$

$$or \sin^n A < 1\ True\ as\ \ \sin A < 1$$