ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC with n_a, n_b, n_c $\to$ Nagel cevians, following relationship holds :

$$n_a + n_b + n_c \geq 2s - 3(2\sqrt{3} - 3)r$$

Proposed by Mohamed Amine Ben Ajiba-Tanger-Morocco

Solution by Soumava Chakraborty-Kolkata-India

$$n_a + n_b + n_c \geq 2s - 3(2\sqrt{3} - 3)r \Leftrightarrow \sum_{cyc} n_a + 6\sqrt{3}r \geq 2s + 9r \overset{\text{squaring}}{\Leftrightarrow}$$

$$\left(\sum_{cyc} n_a\right)^2 + 108r^2 + 12\sqrt{3}r\left(\sum_{cyc} n_a\right) \overset{(*)}{\geq} 4s^2 + 81r^2 + 36rs$$

Now, via Ben Ajiba, $\left(\sum_{cyc} n_a\right)^2 \geq 9s^2 - 80Rr - 2r^2 = 3s^2 + 6(s^2 - 16Rr + 5r^2) +$

$16r(R - 2r) \overset{\substack{\text{Gerretsen}\\ \text{and}\\ \text{Euler}}}{\geq} 3s^2 \Rightarrow 12\sqrt{3}r\left(\sum_{cyc} n_a\right) \geq 36rs$ & so, LHS of $(*)$ − RHS of $(*)$

$\geq 9s^2 - 80Rr - 2r^2 + 108r^2 + 36rs - 4s^2 - 81r^2 - 36rs = 5(s^2 - 16Rr + 5r^2)$

$\overset{\text{Gerretsen}}{\geq} 0 \Rightarrow (*)$ is true $\therefore n_a + n_b + n_c \geq 2s - 3(2\sqrt{3} - 3)r \; \forall \, \Delta$ ABC,

$'' = ''$ iff Δ ABC is equilateral (QED)