ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(h_a \tan\frac{A}{2}\right)^2 + \left(h_b \tan\frac{B}{2}\right)^2 + \left(h_c \tan\frac{C}{2}\right)^2 \le \frac{9R^2}{4}$$

Proposed by George Apostolopoulos-Greece

Solution by Tapas Das-India

$$h_a \tan\frac{A}{2} \overset{h_a \le \sqrt{s(s-a)}}{\le} \sqrt{s(s-a)}\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \sqrt{(s-b)(s-c)} \ (1)$$

$$\left(h_a \tan\frac{A}{2}\right)^2 + \left(h_b \tan\frac{B}{2}\right)^2 + \left(h_c \tan\frac{C}{2}\right)^2 = \sum\left(h_a \tan\frac{A}{2}\right)^2 \overset{(1)}{\le}$$

$$\le \sum (s-b)(s-c) = \sum (s^2 - s(b+c) + bc) =$$

$$= 3s^2 - 2s(a+b+c) + \sum ab = 3s^2 - 4s^2 + s^2 + 4Rr + r^2 =$$

$$= 4Rr + r^2 \overset{Euler}{\le} 4R \cdot \frac{R}{2} + \frac{R^2}{4} = \frac{9R^2}{4}$$

Equality holds for an equilateral triangle