ROMANIAN MATHEMATICAL MAGAZINE

If in Δ ABC, $a \le b \le c$ then :

$$\frac{a+b}{m_a+m_b} \le \frac{b+c}{m_b+m_c}$$

Proposed by Daniel Sitaru-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$m_b + m_c \overset{?}{\le} m_a + m_b \Leftrightarrow 2a^2 + 2b^2 - c^2 \overset{?}{\le} 2b^2 + 2c^2 - a^2$$

$$\Leftrightarrow 3(c^2 - a^2) \overset{?}{\ge} 0 \to \text{true} \because c \ge a \therefore \frac{1}{m_a+m_b} \le \frac{1}{m_b+m_c}$$

$$\Rightarrow \frac{a+b}{m_a+m_b} \le \frac{a+b}{m_b+m_c} \overset{a \le c}{\le} \frac{b+c}{m_b+m_c}$$

$$\therefore \frac{a+b}{m_a+m_b} \le \frac{b+c}{m_b+m_c} \text{ in } \Delta \text{ ABC such that : } a \le b \le c, '' ='' \text{ iff } a = c \text{ (QED)}$$