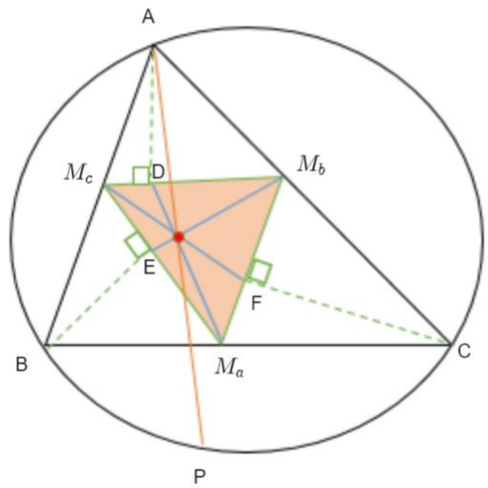
ROMANIAN MATHEMATICAL MAGAZINE



If $\Delta M_aM_bM_c$ is the medial triangle then prove that: DM_a, EM_b, FM_c are in concurrence and

$$AP = \frac{2bc}{\sqrt{-a^2 + 2b^2 + 2c^2}}$$

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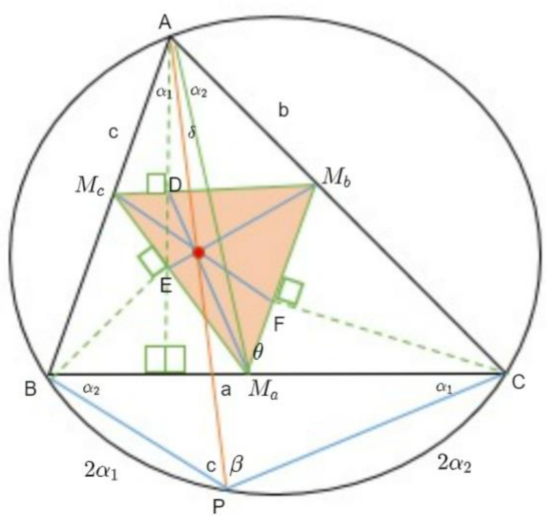

$$Let\ \angle BAP = \alpha_1, \angle CAP = \alpha_2\ and\ \angle AM_aC = \theta; |\alpha_1 - \alpha_2| = \delta$$

$$In\ \Delta ABM_a; \theta = \angle B + \alpha_1 + \delta; \angle BM_aA = \pi - \theta = \angle C + \alpha_2 - \delta \quad (1)$$

$$I\ case:\ if\ \alpha_2 \geq \alpha_1 \Rightarrow \alpha_1 = \alpha_2 - \delta \quad (2)$$

$$In\ \Delta BPC\ rule\ sine\ \frac{BC}{\sin(B+C)}=\frac{PC}{\sin\alpha_2}\Rightarrow\frac{a}{\sin A}=\frac{PC}{\sin\alpha_2}\quad(3)$$

$$In\ \Delta APC\ rule\ sine\ \frac{AP}{\sin(C+\alpha_1)}=\frac{PC}{\sin\alpha_2}\ (4)$$

$$From\ (3)\ and\ (4)\ we\ have\ \frac{a}{\sin A}=\frac{AP}{\sin(C+\alpha_1)}2R\Rightarrow AP=2R\sin(C+\alpha_1)=$$

$$\overset{(2)}{=}2R\sin(C+\alpha_2-\delta)=2R\sin(\pi-\theta)=2R\sin\theta$$

$$In\ \Delta AKM\left(\widehat{K}=90^\circ\right)\Rightarrow\sin\theta=\frac{h_a}{m_a}\ (5)$$

$$AP=2R\sin\theta\overset{(5)}{=}2R\cdot\frac{h_a}{m_a}=\frac{2R\cdot\frac{bc}{2R}}{m_a}=\frac{bc}{m_a}\Rightarrow AP=\frac{bc}{m_a}$$

$$AP=\frac{2bc}{2m_a}=\frac{2bc}{\sqrt{-a^2+2b^2+2c^2}}$$

II case : *if* $\alpha_1\geq\alpha_2$. *The proof is performed analogously as in case I.*

Let us prove that , DM_a, EM_b, FM_c *are concurrent.*

$$\begin{cases}\frac{M_cD}{DM_b}=\frac{c\cos B}{b\cos C}=\frac{2R\sin C\cos B}{2R\sin B\cos C}=\frac{\cot B}{\cot C}\\ \frac{M_bF}{FM_a}=\frac{\cot A}{\cot B};\frac{M_aE}{EM_c}=\frac{\cot C}{\cot A}\end{cases}\Rightarrow\frac{M_cD}{DM_b}\cdot\frac{M_bF}{FM_a}\cdot\frac{M_aE}{EM_c}=1\Rightarrow$$

$\Rightarrow DM_a; EM_b; FM_c$ *are concurrent.*