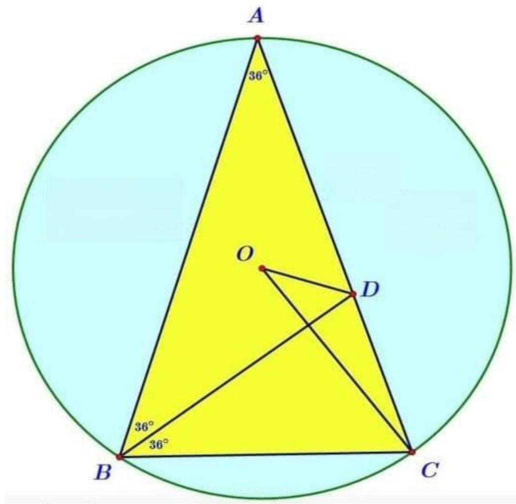
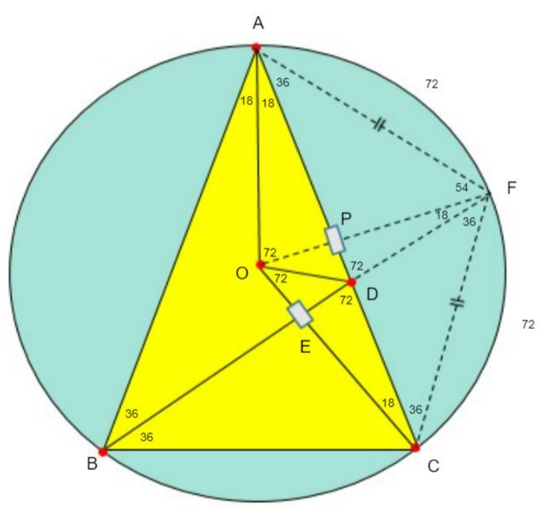
ROMANIAN MATHEMATICAL MAGAZINE

$$Prove\ that:\ \angle COD = 36^\circ$$



Proposed by Jafar Nikpour-Iran

Solution by Mirsadix Muzefferov-Azerbaijan



$$Let's\ do\ construction:\ We\ have\ deltoid\ (kite)\ AOCF.\ Because\ AO = OC = R;$$

$$AF = FC\ (\cup AF = \cup FC).\ Then\ AC \perp OF.\ Also,$$

$$A\widehat{O}F = F\widehat{O}C = 72^\circ;\ A\widehat{F}P = P\widehat{F}C = 54^\circ;$$

$$\Delta AOC\ is\ isosceles.\ Then\ O\widehat{A}C = O\widehat{C}A = 18^\circ, B\widehat{C}E = 54^\circ\ and$$

$$E\hat{B}C = 36° \Rightarrow BE \perp OC \Rightarrow \hat{E} = 90°$$

$Therefore\ E\hat{D}C = 72°\ ;\ A\hat{D}F = E\hat{D}C = 72°\ .\ Then\ O\hat{F}D = 18°\ and\ D\hat{F}C = 36°\ .$

$$Then\ \Delta CDF\ is\ isosceles\ \Rightarrow DF = DC\ .$$

$$\Delta DPF(18°; 72°; DF) \equiv \Delta DEC(18°; 72°; CD) \Rightarrow DP = ED$$

$$\Delta AOP(18°; 72°; R) \equiv \Delta OFE(18°; 72°; R) \Rightarrow OP = OE$$

$$\left.\begin{matrix} OP = OE \\ DP = DE \end{matrix}\right\} and\ OD\ common\ side\ .$$

$$\Rightarrow \Delta OPD = \Delta ODE \Rightarrow$$

$$P\hat{O}D = D\hat{O}E \Rightarrow C\hat{O}D = 36°\ \ (E\hat{O}P = 72°)$$