ROMANIAN MATHEMATICAL MAGAZINE

Let $a,b,c\geq 0, a+b+c=4$. Prove that:

$$\frac{a}{a+4b+2}+\frac{b}{b+4c+2}+\frac{c}{c+4a+2}\leq\frac{2}{3}\qquad(*)$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$(*)\Leftrightarrow\frac{3a}{a+4b+2}+\frac{3b}{b+4c+2}+\frac{3c}{c+4a+2}\leq 2=\frac{a}{6-a}+\frac{3(b+c)}{6-a}$$

$$\Leftrightarrow a\left(\frac{1}{6-a}-\frac{3}{a+4b+2}\right)+3b\left(\frac{1}{6-a}-\frac{1}{b+4c+2}\right)+3c\left(\frac{1}{6-a}-\frac{1}{c+4a+2}\right)\geq 0$$

$$\Leftrightarrow -\frac{4ac}{(6-a)(a+4b+2)}+\frac{9bc}{(6-a)(b+4c+2)}+\frac{3c(4a-b)}{(6-a)(c+4a+2)}\geq 0$$

$$\Leftrightarrow 4ca\left(\frac{3}{c+4a+2}-\frac{1}{a+4b+2}\right)+3bc\left(\frac{3}{b+4c+2}-\frac{1}{c+4a+2}\right)\geq 0$$

$$\Leftrightarrow\frac{52abc}{(c+4a+2)(a+4b+2)}+\frac{52abc}{(b+4c+2)(c+4a+2)}\geq 0,$$

which is true and the proof is complete.

Equality holds if $a=b=2, c=0$ and its permutations.