ROMANIAN MATHEMATICAL MAGAZINE

Let $a,b,c\ge 0, ab+bc+ca>0$. Prove that:

$$\frac{a(b+c)}{ab+ac+4bc}+\frac{b(c+a)}{bc+ba+4ca}+\frac{c(a+b)}{ca+cb+4ab}\le\frac{a^2+b^2+c^2}{ab+bc+ca}$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$(ab+bc+ca)\sum_{cyc}\frac{a(b+c)}{ab+ac+4bc}=\sum_{cyc}\left(a(b+c)-\frac{3abc(b+c)}{ab+ac+4bc}\right)\le$$

$$\overset{AM-GM}{\le} 2(ab+bc+ca)-3abc\sum_{cyc}\frac{b+c}{ab+ac+(b+c)^2}=$$

$$=2(ab+bc+ca)-\frac{9abc}{a+b+c}\overset{Schur}{\le} a^2+b^2+c^2$$

which completes the proof.

Equality holds iff $a=b=c=1$ & $a=b, c=0$ and permutations.