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THE ENDLESS DESERT OF DECIMALS OF PI - RESULTS WITHOUT WORDS

By Neculai Stanciu-Romania

I. The perimeter of the circle with diameter equal to unity is equal to π.

I.1. The perimeter of the regular n-sided polygon inscribed in the circle of radius R is $p_n = 2nR\sin\frac{\pi}{n}$.

For $2R = 1 \Rightarrow p_n = n\sin\frac{\pi}{n}$.

$p_3 = 3\sin\frac{\pi}{3} =$ 2,598076211...;

$p_4 = 4\sin\frac{\pi}{4} =$ 2,828427124...;

$p_6 = 6\sin\frac{\pi}{6} =$ **3**;

$p_{57} = 57\sin\frac{\pi}{57} =$ **3,14**0002340...;

$p_{94} = 94\sin\frac{\pi}{94} =$ **3,141**007838...;

....

$p_{2022} = 2022\sin\frac{\pi}{2022} =$ **3.14159**138962198602512416672271708031346771...

$p_{2023} = 2023\sin\frac{\pi}{2023} =$ **3.14159**139087127446356375816093428982806031...

$p_n = n\sin\frac{\pi}{n} < \pi$ $(2R = 1)$; if $n \to \infty$, then $p_n \to \pi$.

$p_n = 2nR\sin\frac{\pi}{n}$, if $n \to \infty$, then $p_n \to 2\pi R$.

I.2. The perimeter of the regular polygon with n sides circumscribed in the circle of radius r is $P_n = 2nrtg\frac{\pi}{n}$.

For $2r = 1 \Rightarrow P_n = ntg\frac{\pi}{n}$.

$P_3 = 3tg\frac{\pi}{3}$=5,196152422…;

$P_4 = 4tg\frac{\pi}{4}$=4;

$P_6 = 6tg\frac{\pi}{6}$=**3**,464101615…;

$P_{36} = 36tg\frac{\pi}{36}$=**3,14**9591886…;

$P_{160} = 160tg\frac{\pi}{160}$=**3,141**996443…;

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$P_{2022} = 2022tg\frac{\pi}{2022}$=**3.14159**5181528153765416466771500082094511498951…

$P_{2023} = 2023tg\frac{\pi}{2023}$=**3.14159**5179029571462801308508332406982889593998…

$2r = 1$, $P_n = ntg\frac{\pi}{n} > \pi$; if $n \to \infty$, then $P_n \to \pi$.

$P_n = 2nrtg\frac{\pi}{n}$, if $n \to \infty$, then $P_n \to 2\pi r$.

II. The area of the circle with radius equal to unity is equal to π.

II.1. The area of the regular polygon with n sides inscribed in the circle of radius R is $\Delta_n = \frac{nR^2}{2}\sin\frac{2\pi}{n}$.

For $R = 1 \Rightarrow \Delta_n = \frac{n}{2}\sin\frac{2\pi}{n}$.

$\Delta_3 = \frac{3}{2}\sin\frac{2\pi}{3}$ =1,299038105…;

$\Delta_4 = \frac{4}{2}\sin\frac{2\pi}{4}$ = 2;

$\Delta_6 = \frac{6}{2}\sin\frac{2\pi}{6}$ =2,598076211…;

$\Delta_{114} = \frac{114}{2}\sin\frac{2\pi}{114}$ =**3,14**0002340…;

$\Delta_{187} = \frac{187}{2}\sin\frac{2\pi}{187}$ =**3,141**001567…;

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$\Delta_{2022} = 1011\sin\frac{\pi}{1011}$ =**3.1415**87597720395116522177825730598930403099 2…;

$\Delta_{2023} = \frac{2023}{2}\sin\frac{2\pi}{2023}$ = **3.1415**8760271754525312946058535116446105443 8…

$R = 1$, $\Delta_n = \frac{n}{2}\sin\frac{2\pi}{n} < \pi$; then for $n \to \infty$ we have $\Delta_n \to \pi$.

$\Delta_n = \frac{nR^2}{2}\sin\frac{2\pi}{n}$; if $n \to \infty$, then $\Delta_n \to \pi R^2$.

II.2. The area of the regular polygon with n sides circumscribed in the circle of radius r is $A_n = nr^2 tg\frac{\pi}{n}$.

For $r = 1 \Rightarrow A_n = ntg\frac{\pi}{n}$.

$A_3 = 3tg\frac{\pi}{3}$=5,196152422…;

$A_4 = 4tg\frac{\pi}{4}$=4;

$A_6 = 6tg\frac{\pi}{6}$=**3,**464101615…;

$A_{36} = 36tg\frac{\pi}{36}$=**3,14**9591886…;

$A_{160} = 160tg\frac{\pi}{160}$=**3,141**996443…;

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$A_{2022} = 2022tg\frac{\pi}{2022}$=**3.14159**5181528153765416466771500082094511498951…;

$A_{2023} = 2023tg\frac{\pi}{2023}$=**3.14159**5179029571462801308508332406982889593998…

$r=1$, $A_n = ntg\frac{\pi}{n} > \pi$; for $n \to \infty$ we have $A_n \to \pi$.

$A_n = nr^2 tg\frac{\pi}{n}$; if $n \to \infty$, then $A_n \to \pi r^2$.

Remark. The best results are obtained when calculating the perimeter of regular polygons inscribed in the circle with a diameter equal to the unit. In the case of circumscribed polygons, the results for perimeter and area are identical.