ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro

ABOUT A FEW SPECIAL TRIANGLES

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Abstract: *In this paper are presented new inequalities in triangle and consequences.*

1. Introduction

We will study this questions:

$\frac{n_a}{h_a}, \frac{n_b}{h_b}, \frac{n_c}{h_c}$ **can be sides in triangle ?**

an_a, bn_b, cn_c **can be sides in triangle ?**

$\frac{r_a}{s+n_a}, \frac{r_b}{s+n_b}, \frac{r_c}{s+n_c}$ **can be sides in triangle ?**

$\frac{r_a}{s-n_a}, \frac{r_b}{s-n_b}, \frac{r_c}{s-n_c}$ **can be sides in triangle ?**

In $\Delta ABC, N_a$ −Nagel's point and n_a −Nagel's cevian:

$$AN_a = \frac{a \cdot n_a}{s} \text{ and } BN_a + CN_a \geq BC \ (triangle\ inequality)$$

$$\frac{b \cdot n_b}{s} + \frac{c \cdot n_c}{s} \geq a \Rightarrow b \cdot n_b + c \cdot n_c \geq a \cdot s$$

$$2F = a \cdot h_a = b \cdot h_b = c \cdot h_c = 2sr$$

2. Main result

$$\frac{n_b}{h_b} + \frac{n_c}{h_c} \geq \frac{a}{2r}; \qquad (1)$$

$$s^2 = n_a^2 + 2r_a h_a \Rightarrow s^2 - n_a^2 = 2r_a h_a$$

$$(s+n_a)(s-n_a)=2r_ah_a \Rightarrow s-n_a=\frac{2r_ah_a}{s+n_a}$$

$$s=n_a+\frac{2r_ah_a}{s+n_a} \Rightarrow \frac{s}{h_a}=\frac{n_a}{h_a}+\frac{2r_a}{s+n_a} \text{ and } \frac{s}{n_a}=\frac{a}{2r}$$

$$\Rightarrow \frac{a}{2r}=\frac{n_a}{h_a}+\frac{2r_a}{s+n_a}; \qquad (2)$$

From (1) and (2), we have:

$$\frac{n_b}{h_b}+\frac{n_c}{h_c}\geq\frac{n_a}{h_a}$$

$\frac{n_a}{h_a},\frac{n_b}{h_b},\frac{n_c}{h_c}$ – can be lengths of sides of a triangle, then

an_a, bn_b, cn_c –can be lengths of sides of a triangle.

If a,b,c are lengths sides of a triangle, then $\sqrt{a},\sqrt{b},\sqrt{c}$ are lengths sides of acute triangle.

So, $\sqrt{\frac{n_a}{h_a}},\sqrt{\frac{n_b}{h_b}},\sqrt{\frac{n_c}{h_c}}$ are lengths of sides of acute triangle, and

$\sqrt{an_a},\sqrt{bn_b},\sqrt{cn_c}$ are lengths sides of acute triangle.

$$h_a=\frac{2r_br_c}{r_b+rc} \Rightarrow \frac{1}{h_a}=\frac{r_b+r_c}{2r_br_c} \Rightarrow \frac{2}{h_a}=\frac{1}{r_b}+\frac{1}{r_c}$$

$$\frac{2s}{h_a}=\frac{s}{r_b}+\frac{s}{r_c}; 2F=ah_a=2sr \Rightarrow \frac{2s}{h_a}=\frac{a}{r}$$

$$\begin{cases}\cot\frac{B}{2}=\frac{s}{r_b}\\ \cot\frac{C}{2}=\frac{s}{r_c}\end{cases} \Rightarrow \frac{a}{r}=\cot\frac{B}{2}+\cot\frac{C}{2} \text{ and } a+b>c$$

$$\begin{cases}x_1=\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{a}{r}\\ y_1=\cot\frac{A}{2}+\cot\frac{C}{2}=\frac{b}{r}\\ z_1=\cot\frac{A}{2}+\cot\frac{C}{2}=\frac{c}{r}\end{cases} \Rightarrow x_1+y_1>z_1(\text{ and analogs})$$

So, x_1,y_1,z_1 can be lengths sides of a triangle.

$$s^2=n_a^2+2r_ah_a \Rightarrow s^2-n_a^2=2r_ah_a$$

$$(s+n_a)(s-n_a)=2r_ah_a \Rightarrow s-n_a=\frac{2r_ah_a}{s+n_a}$$

$$s=n_a+\frac{2r_ah_a}{s+n_a}\Rightarrow\frac{s}{h_a}=\frac{n_a}{h_a}+\frac{2r_a}{s+n_a}\Rightarrow\frac{a}{2r}=\frac{n_a}{h_a}+\frac{2r_a}{s+n_a}$$

$$\frac{1}{2}(y_1+z_1)>\frac{1}{2}x_1\Leftrightarrow\frac{b}{2r}+\frac{c}{2r}>\frac{a}{2r}\Leftrightarrow$$

$$\frac{n_b}{h_b}+\frac{n_c}{h_c}+\frac{2r_b}{s+n_b}+\frac{2r_c}{s+n_c}>\frac{n_a}{h_a}+\frac{2r_a}{s+n_a}$$

$$\text{But}:\frac{n_b}{h_b}+\frac{n_c}{h_c}>\frac{n_a}{h_a},\text{then}:\frac{r_b}{s+n_b}+\frac{r_c}{s+n_c}>\frac{r_a}{s+n_a}$$

So, $\frac{r_a}{s+n_a},\frac{r_b}{s+n_b},\frac{r_c}{s+n_c}$ can be lengths sides of a triangle, then:

$\sqrt{\frac{r_a}{s+n_a}},\sqrt{\frac{r_b}{s+n_b}},\sqrt{\frac{r_c}{s+n_c}}$ can be lengths sides of acute triangle.

$$s^2-n_a^2=2r_ah_a\Rightarrow(s-n_a)(s+n_a)=2r_ah_a$$

$$s+n_a=\frac{2r_ah_a}{s-n_a}\Rightarrow\frac{s}{h_a}+\frac{n_a}{h_a}=\frac{2r_a}{s-n_a}\Rightarrow\frac{2r_a}{s-n_a}=\frac{a}{2r}+\frac{n_a}{h_a}$$

Therefore, $\frac{r_a}{s-n_a},\frac{r_b}{s-n_b},\frac{r_c}{s-n_c}$ can be lengths sides of a triangle.

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