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A SIMPLE PROOF FOR KLAMKIN'S INEQUALITY

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Abstract: *In this short math note is presented a new technique to proof Klamkin's inequality.*

In $\Delta ABC, P, P' \in (ABC)$ the following relationship holds:

$$a \cdot AP \cdot AP' + b \cdot BP \cdot BP' + c \cdot CP \cdot CP' \geq abc \ (Klamkin)$$

Proof.

For $x, y, z \in \mathbb{R}$ we have:

$$(x+y+z)(x \cdot AP^2 + y \cdot BP^2 + z \cdot CP^2) \geq yz \cdot a^2 + zx \cdot b^2 + xy \cdot c^2$$

Let $P' \in Int(ABC)$ and $x = \frac{a \cdot AP'}{AP}, y = \frac{b \cdot BP'}{BP}, z = \frac{c \cdot CP'}{CP}$. Hence,

$$\left(\sum_{cyc} \frac{a \cdot AP'}{AP}\right) \cdot \left(\sum_{cyc} \frac{a \cdot AP'}{AP} \cdot AP^2\right) \geq \sum_{cyc} \frac{bc \cdot BP' \cdot CP'}{BP \cdot CP} \cdot a^2$$

$$\frac{1}{AP \cdot BP \cdot CP} \sum_{cyc} a \cdot AP' \cdot BP \cdot CP \cdot \sum_{cyc} a \cdot AP' \cdot AP \geq \frac{abc}{AP \cdot BP \cdot CP} \sum_{cyc} a \cdot AP \cdot BP' \cdot CP'$$

$$\sum_{cyc} a \cdot AP' \cdot BP \cdot CP \cdot \sum_{cyc} a \cdot AP' \cdot AP \geq abc \sum_{cyc} a \cdot AP \cdot BP' \cdot CP'; (1)$$

Analogous,

$$\sum_{cyc} a \cdot AP \cdot BP' \cdot CP' \cdot \sum_{cyc} a \cdot AP \cdot AP' \geq abc \sum_{cyc} a \cdot AP' \cdot BP \cdot CP; (2)$$

By adding, it follows that:

$$\sum_{cyc} a \cdot AP \cdot AP' \left(\sum_{cyc} a \cdot AP' \cdot BP \cdot CP + \sum_{cyc} a \cdot AP \cdot BP' \cdot CP'\right) \geq$$

$$\geq abc \left(\sum_{cyc} a \cdot AP \cdot BP' \cdot CP' + \sum_{cyc} a \cdot AP \cdot BP' \cdot CP'\right)$$

$$a \cdot AP \cdot AP' + b \cdot BP \cdot BP' + c \cdot CP \cdot CP' \geq abc \ (Klamkin)$$

REFERENCE:

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