PP38296

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In all triangles ABC holds:

1. $$\sum_{cyc} m_a^3 w_a r_a \geq \frac{3}{2} s^2 r(s^2 - r^2 - 4Rr)$$

2. $$\sum_{cyc} m_a w_a r_a^2 \geq (4R + r)s^2 r$$

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Solution by Daniel Sitaru.

1. $$\sum_{cyc} m_a^3 w_a r_a = \sum_{cyc} m_a^2 \cdot m_a w_a \cdot r_a \geq$$

$$\geq \sum_{cyc} m_a^2 \cdot s(s-a) \cdot \frac{F}{s-a} =$$

$$= sF \sum_{cyc} m_a^2 = sF \cdot \frac{3}{4}(a^2 + b^2 + c^2) =$$

$$= s \cdot rs \cdot \frac{3}{4} \cdot 2(s^2 - r^2 - 4Rr) = \frac{3}{2} rs^2(s^2 - r^2 - 4Rr)$$

2. $$\sum_{cyc} m_a w_a r_a^2 \geq \sum_{cyc} s(s-a) \cdot \frac{F^2}{(s-a)^2} =$$

$$= sF^2 \cdot \sum_{cyc} \frac{1}{s-a} = s \cdot r^2 s^2 \cdot \frac{4R + r}{rs} =$$

$$= rs^2(4R + r)$$

□

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